Math 10A
Worksheet, Discussion #28; Wednesday, 7/25/2018
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1 First-Order ODEs

1.1 Concepts

1. Integrating factors is for **linear, first-order** differential equations. It is of the form

$$\boxed{y' + P(t)y = Q(t).}$$

To solve this, we multiply the equation by the **integrating factor** $I(t) = e^{\int P(t)dt}$. Then we can calculate that $I'(t) = I(t)P(t)$ and

$$(I(t)y)' = I(t)y' + I'(t)y = I(t)y' + I(t)P(t)y = I(t)(y' + P(t)y) = I(t)Q(t).$$

Taking the integral gives us

$$I(t)y = \int I(t)Q(t)dt \implies y = \frac{1}{I(t)}\int I(t)Q(t)dt.$$

2. Separable equations is for **first-order** differential equations which can be separated. It is **separable** if we can write it of the form

$$\boxed{y' = f(y)g(t).}$$

We write $y' = \frac{dy}{dt}$ and split it to get

$$\int \frac{dy}{f(y)} = \int g(t)dt.$$

If $f(y)$ is a polynomial, we use **partial fractions** to integrate the left side. There are also solutions gotten by solving $f(y) = 0$.

1.2 Problems

3. True False We cannot use the method of separable equations on $y' = e^{y+t}$ because it involves a sum of y and t.

4. True False If we can use the method of separable equations, we must be able to write $y' = (ay + b)f(t)$ for a linear polynomial in terms of y.

5. True False The equation $y' = y + t$ is not separable and so we do not know how to solve it.

6. Find the solution of $y' + 2xy = 2x$ with $y(0) = 0$.

7. Find the general solution to $y' - \frac{y}{x+1} = (x+1)^2$.

8. Find the solution to $\dfrac{dy}{dt} = 2y + 3$ with $y(0) = 0$.

9. Find the solution to $y'e^y = 2t + 1$ with $y(1) = 0$.

10. Solve the IVP $y' = te^t$ with $y(0) = 0$.

11. Solve the IVP $y' = \frac{1}{t \ln t}$ with $y(e) = 0$.